Surface Stress over the Ocean in Swell-Dominated Conditions during Moderate Winds

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ABSTRACT

Atmospheric and surface wave data from several oceanic experiments carried out on the Floating Instrument Platform (FLIP) and the Air–Sea Interaction Spar (ASIS) have been analyzed with the purpose of identifying swell-related effects on the surface momentum exchange during near-neutral atmospheric conditions and wind-following or crosswind seas. All data have a pronounced negative maximum in \( u_w \cos(\omega) \) centered at the frequency of the dominant swell \( n_p \), meaning a positive contribution to the stress. A similar contribution at this frequency is also obtained for the corresponding crosswind cospectrum. The magnitude of the cospectral maximum is shown to be linearly related to the square of the orbital motion, being equal to \( \frac{H^2_{sd} \omega^2}{n_p^2} \), where \( H_{sd} \) is the swell-significant wave height, the effect tentatively being due to strong correlation between the surface component of the orbital motion and the pattern of capillary waves over long swell waves.

A model for prediction of the friction velocity \( u_* \) from measurements of \( H_{sd}, n_p, \) and the 10-m wind speed \( U_{10} \) is formulated and tested against an independent dataset of \( \approx 400 \) half-hour measurements during swell, giving good result.

The model predicts that the drag coefficient \( C_D \), which is traditionally modeled as a function of \( U_{10} \) alone (e.g., the COARE algorithm), becomes strongly dependent on the magnitude of the swell factor \( H^2_{sd} \omega^2 \) and that \( C_D \) can attain values several times larger than predicted by wind speed–only models. According to maps of the global wave climate, conditions leading to large effects are likely to be widespread over the World Ocean.

1. Introduction

Most current parameterizations of the surface drag over the sea simply take the drag coefficient \( C_D = \frac{u_*^2}{U_{10}^2} \), where \( u_* \) is friction velocity and \( U_{10} \) is the wind speed at 10 m, to be a function of wind speed [e.g., the Coupled Ocean–Atmosphere Response Experiment (COARE 3.0) algorithm (Fairall et al. 2003)]. But plots of individual data points reveal very large scatter (see, e.g., Edson 2008), in particular in the wind speed range below about 7 m s\(^{-1}\). The rationale behind the choice of a single wind speed–dependent expression is that the data scatter is assumed simply to reflect noise. But it is a well-known fact (see, e.g., Semedo et al. 2011) that swell prevails over most of the World Ocean, and, as seen below, there is evidence available in the literature that the surface stress, in particular in the low-to-medium wind speed range, may be strongly linked to it. As swell-related effects may depend on other factors than wind speed, this may contribute significantly to the observed scatter in plots of \( C_D \) against wind speed.

In a recent study (Högström et al. 2013), a detailed comparison was made of atmospheric data obtained in swell-dominated conditions at a site in the Baltic Sea (Östergarnsholm) and at a true oceanic site outside Hawaii. Data are, respectively, from the Baltic Sea Swell
Experiment (BASE) and the Rough Evaporation Duct (RED) experiment, details of which will be given later (datasets available upon request). An interesting result from this study was the surprisingly good agreement of several features at the two sites: 1) strong increase in wind speed in a shallow layer above the water surface (7–8 m deep) followed by virtual constancy of the wind above, with a consequent nonvalidity of Monin–Obukhov (MO) theory, and 2) strong resemblance of the height variation of all the terms of the turbulence kinetic energy budget. In strong contrast to these results, the drag coefficient $C_D = u^2/\delta$ plotted against $U_{10}$ was found to behave very differently at the two sites during swell conditions in the approximate wind speed range 3–7 m s$^{-1}$, (see Fig. 1). Thus, the curve denoted BASE swell is systematically lower than the corresponding curve denoted RED. In that graph are also given data from two other oceanic experiments: The Tropical Ocean Global Atmosphere (TOGA) COARE and the San Clemente Ocean Probing Experiment (SCOPE), derived from plots in Grachev and Fairall (2001). It is seen that the RED data agree with the TOGA COARE data down to 3 m s$^{-1}$, which is the lowest wind speed in the RED data. The SCOPE data are found to be intermediate to the BASE and the TOGA COARE–RED data. Note that, in Fig. 1, $C_D$ is used and not $C_{DN}$ because, as argued in Högström et al. (2013), MO theory is invalid in swell conditions.

In several recent papers (e.g., Sullivan et al. 2008; Högström et al. 2009; Hanley and Belcher 2008), it has been demonstrated how form drag over swell waves can produce upward-directed momentum flux, which would mean negative $C_D$. A prerequisite for this to be observed over the ocean is the occurrence of low wind speed, as, in fact, reported to occur in the SCOPE and TOGA COARE datasets for winds below about 1.5 m s$^{-1}$ (Grachev and Fairall 2001) and low values of the inverse wave age parameter $U_{10}/c_p$ (Hanley and Belcher 2008) and as demonstrated in a forthcoming paper based on data from the “flux, état de la mer, et télédétection en condition de fetch variable” (FETCH). The strong increase found in Fig. 1 for $C_D$ in the 3–6 m s$^{-1}$ range during swell for the RED, SCOPE, and TOGA COARE datasets, compared to the corresponding BASE dataset, appears to be linked to the magnitude of the swell in some up-to-now unknown way (the significant wave height of the former three being in the range 1.0–2.0 m and that of BASE typically being 0.3 m).

Rieder (1997) analyzed turbulence measurements performed onboard the Research Platform (R/P) Floating Instrument Platform (FLIP) during an experiment [the Surface Waves Processes Program (SWAPP)], which was carried out 500 km off the California coast and lasted 22 days. To the author’s disappointment, he did not manage to find any statistically satisfactory relation between the drag coefficient and the wind speed in this dataset. His conclusion was that “waves from different parts of the ocean create conditions in which the waves often have little to do with the local wind” and that “clearly, these conditions, in which a mixture of waves coexist, are the most poorly understood; however, these are the conditions that are most common globally and therefore of greatest importance” (Rieder 1997).

In a follow-up paper a year later, Rieder and Smith (1998) analyzed another FLIP dataset, the Marine Boundary Layer Experiment (MBLEX) which was taken from a 4-day period in February–March 1995, when the platform was freely drifting about 100 km off the California coast. The authors divide the atmospheric stress cospectra in three frequency bands, defined in their own words as: “(1) low, below significant surface wave energy, defined as below 0.06 Hz; (2) middle or ‘wave band’ where correlations with surface elevations can be expected to show up at the 8-m anemometer height, 0.06–0.16 Hz; and (3) high, corresponding to wavelengths too short to maintain direct correlation at 8 m” (Rieder and Smith 1998). In order to “isolate the direct wave influence,” the authors use the complex cross-spectra between horizontal $u'$, $v'$ and vertical $w'$ fluctuating air velocities and the sea surface elevation.

In their Fig. 5, the authors show spectra from two selected cases: 1900 UTC 7 March and 1200 UTC 8 March. Each plot contains, in log–log representation,
wave spectra and stress density spectra at 8 m, both total spectra and residual spectra, obtained after removal of the wave-correlated part. The atmospheric spectra from those graphs have been replotted here in a representation with $nC_{uw}(n)$ on a linear scale and frequency on a logarithmic scale: Figs. 2 and 3, respectively. Since we have $d(ln)C_{uw}(n) = dnC_{uw}(n)$, this representation is area preserving. From Figs. 2 and 3, it is evident that there is a strong negative peak at the dominant frequency (about 0.08 Hz) and that the wave-correlated filter apparently removes this peak. The wind speed in both cases is low ($\leq 2.5$ m s$^{-1}$ in Fig. 2 and $\leq 5.5$ m s$^{-1}$ in Fig. 3), implying that waves with frequency 0.08 Hz cannot be caused by the local wind forcing but must be swell.

Cospectral peaks at the frequency of the dominant swell $n_p$, similar to those in Figs. 2 and 3, are observed in all cospectra obtained during swell conditions for the experiment RED and other swell-dominated data presented below. Rieder and Smith (1998) found that, when they removed the wave-correlated part of the cospectrum, the residual spectral energy could be expressed as a function of the 10-m wind speed alone. This would represent the wind-sea contribution to the momentum flux. The approach of the present paper is as follows: 1) to identify and evaluate swell-related peaks in atmospheric velocity cospectra from several oceanic data-sets; 2) to subtract these peak contributions from the corresponding total drag coefficient and compare the resulting residual with a wind-sea parameterization of drag; 3) to formulate a theory for the swell peak contribution; and 4) to formulate a general model for the drag in situations with following or crosswind swell (see below for a definition) and near-neutral atmospheric stratification.

2. Datasets

a. Data selection

The data are taken from the following sources:

1) Rieder and Smith (1998), the two cases displayed in their Fig. 5 and replotted here in lin–log representation in Figs. 2 and 3
2) Data from the RED experiment (Anderson et al. 2004; see also Högström et al. 2013)
3) Data from the BASE experiment (Högström et al. 2008)
4) Data from the Southern Ocean Gas Exchange Experiment (SOGASEX; Sahlée et al. 2012)

The following general selection criteria have been applied:

(i) Near-neutral atmospheric stratification, defined here as $z_m/L_{MO} < 0.1$, where $z_m$ is measurement height, and $L_{MO}$ is the Obukhov length:

$$L_{MO} = -\frac{u^3_0 T_0}{gkw\theta_v}.$$  \hspace{1cm} (1)

where $T_0$ is a mean layer temperature (K), $g$ is the acceleration due to gravity (m s$^{-2}$), $w\theta_v$ is the
kinematic vertical flux of virtual potential temperature \((\text{m s}^{-1} \text{K})\), and \(u_\theta\) is the friction velocity \((\text{m s}^{-1})\), which is derived from the magnitude of the total stress vector \(\tau\) through the relation

\[
\tau \rho = u_\theta^2 = \sqrt{(\bar{u}'w')^2 + (\bar{v}'w')^2},
\]

where \(\rho\) is air density.

(ii) Prevailing swell conditions, defined as \(c_p/U_{10} > 1.2\), where \(c_p\) is the phase speed of the dominant swell

(iii) Wind-following or crosswind swell conditions (i.e., the angle between the wind and the swell propagation direction \(|\phi| < 90^\circ\); as shown in Högbom et al. (2011) and in Appendix A, the effect of such \(\phi\) values is expected to differ little from the case \(\phi = 0\).

In addition to these three general criteria, we add a fourth criterion at this stage of the analysis:

(iv) Reasonably steady conditions during a number \(N\) of consecutive 30-min periods. The range of \(N\) in the present dataset is \(2 < N < 16\), with a mean of 7 (Table 1). For screening purposes, the wind speed at around \(z = 10\text{ m}\), \(U_{10}\) has been used, and the selection criteria has been absence of a trend in \(U_{10}\) during the period selected. This includes both linear trends and curved trends.

The reason for adding criterion (iv) is that we want to reduce possible effects from nonstationarity in the exploratory phase of the present analysis. In all, 18 cases were selected. Table 1 gives an overview of prevailing meteorological and surface wave characteristics. It is seen that a wide range of conditions are represented: significant wave height \(H_s\) ranging from 0.7 to 4.2 m, wind speed from 2.5 to 12.5 m s\(^{-1}\), peak swell frequency from 0.084 to 0.173 Hz, and wave age \(c_p/U_{10}\) from 1.3 to 7.3. The swell-significant wave height \(H_{sd}\) which ranges from 0.62 to 3.69 m, has been determined as \(H_{sd} = 4(E_{\text{swell}})^{1/2}\), where \(E_{\text{swell}}\) is the energy of the swell waves determined by integration of the wave spectrum

\[
E_{\text{swell}} = \int_{n_0}^{n} S(n) \, dn,
\]

where \(n_0\) is defined by the requirement that the corresponding wave phase speed \(c_0 = 1.14U_{10}\) giving \(n_0 = 0.88g/2\pi U_{10}\). When the spectrum is unknown, \(E_{\text{swell}}\) can be approximated by subtracting the wind-sea component from the total spectrum:

\[
E_{\text{swell}} = E_{\text{tot}} - E_{\text{windsea}}
\]

where \(E_{\text{windsea}}\) is approximated by \(10^{-3}U_{10}^4/g^2\), which can be derived from spectrum \(S(n) = \alpha_{ag}U_{10}^2(2\pi)^{-3}n^{-4}\), \(n > n_0\) using \(\alpha_{ag} = 4.5 \times 10^{-3}\) (Kahma 1981).

From the analysis based on the 18 cases of Table 1, a model for predicting the surface stress during swell conditions was formulated in section 6, where it was validated against an independent dataset of about 400 half-hour data points.

The dataset defined with the criteria (i)–(iv) contains some cases with \(U_{10} < 3.5\text{ m s}^{-1}\). Since analysis shows that these cases have properties that clearly distinguish them from the remaining data, further analysis of these...
cases is preferably done within a wider context of lowwind cases, as will be done in a forthcoming paper.

b. Short characterization of the measurements

The Rieder and Smith (1998) data are taken from measurements with a sonic anemometer at 8-m height above the water onboard the FLIP during an experiment (MBLEX) carried out in the Pacific about 100 km from the California coast. Note that the Rieder and Smith (1998) data are only used here for a background study and not in the quantitative evaluation.

The RED data were also obtained from measurements onboard FLIP, but with sonic anemometers at four levels: 5.1, 6.9, 9.9, and 13.8 m, respectively. The measurements were taken at a site 10 km northeast of the Hawaiian island of Oahu in about 370 m of water. The measurements were conducted from late August to mid-September 2001. The meteorological conditions were characterized by a northeasterly trade wind and with swell present during most of the time. Further details about the measurements are found in Högström et al. (2013).

The BASE measurements were carried out in the Baltic Sea, with instrumentation at several levels (sonic anemometers at 10, 18, and 26 m above mean sea level, as well as slow response sensors for wind speed, direction, and temperature on five levels) on a 30-m-high tower placed at the southernmost tip of the low island Östergarnsholm and, simultaneously, at two levels (2.6 and 5.1 m) on an ASIS buoy (Graber et al. 2000) anchored 4 km south of the tower during September and October 2003 (Högström et al. 2008). Swell was occurring about 30% of the time, but the significant wave height was mostly low (typically 0.3 m or less). This means that cospectral peaks were difficult to identify and evaluate with satisfactory accuracy most of the time. For the present analysis, data from only one period with unusually high swell, significant wave height ($H_{\text{sw}} = 0.62$ m, see Table 1 case BA) were used.

Data from the SOGASEX were taken from Sahlée et al. (2012). The experiment was conducted in the Southern Ocean at around latitude 50°S. Measurements were obtained with a sonic anemometer on an ASIS buoy at 4.5 m above the water surface. The buoy was freely drifting for about 8 days. Conditions were highly variable, but eight cases were selected that fulfill the criteria stated in section 2a, four of which (Si1–Si4) are from slightly unstable conditions and another four of which (Ss1–Ss4) are from slightly stable conditions.

### 3. Height variation of swell-related spectral peaks

All cospectra for the 18 selected cases show distinct peaks at the respective swell frequency in the manner demonstrated for the Rieder and Smith (1998) cases in

<table>
<thead>
<tr>
<th>N</th>
<th>$H_u$ (m)</th>
<th>$H_{\text{sd}}$ (m)</th>
<th>Case</th>
<th>$u_\text{aw}$ (m s$^{-1}$)</th>
<th>$U_{10}$ (m s$^{-1}$)</th>
<th>$n_p$ (s$^{-1}$)</th>
<th>$\kappa_p$ (m$^{-1}$)</th>
<th>$c_p/U_{10}$</th>
<th>$L_{\text{MO}}$ (m)</th>
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<tr>
<td>6</td>
<td>2.60</td>
<td>2.00</td>
<td>R&amp;S 1</td>
<td>0.137</td>
<td>2.5</td>
<td>0.085</td>
<td>0.029</td>
<td>7.3</td>
<td>—</td>
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<tr>
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<td>2.50</td>
<td>1.80</td>
<td>R&amp;S 2</td>
<td>0.16</td>
<td>5.50</td>
<td>0.085</td>
<td>0.029</td>
<td>3.3</td>
<td>—</td>
</tr>
<tr>
<td>16</td>
<td>1.37</td>
<td>1.36</td>
<td>RA</td>
<td>0.136</td>
<td>3.18</td>
<td>0.118</td>
<td>0.056</td>
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</tr>
<tr>
<td>8</td>
<td>1.92</td>
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<td>RB</td>
<td>0.279</td>
<td>7.98</td>
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<td>1.9</td>
<td>—132</td>
</tr>
<tr>
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<td>1.18</td>
<td>1.08</td>
<td>RC</td>
<td>0.199</td>
<td>5.70</td>
<td>0.122</td>
<td>0.060</td>
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<tr>
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<td>1.07</td>
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</tr>
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<td>3.46</td>
<td>Ss1</td>
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<td>0.034</td>
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<td>2.08</td>
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<td>0.038</td>
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<td>80</td>
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<tr>
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<td>Ss4</td>
<td>0.523</td>
<td>10.30</td>
<td>0.093</td>
<td>0.034</td>
<td>1.63</td>
<td>850</td>
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</table>

Table 1. Overview of data. The cases are as follows: R&S 1 and 2 are derived from Fig. 5 of Rieder and Smith (1998), 1900 UTC 7 Mar and 1200 UTC 8 Mar, respectively (measurement height = 8 m); cases RA–RC and RE–RH are from the RED experiment (5 m); case BA is from BASE, 1700 UTC 20 Sep 2003 (2.6 and 5 m); cases Si1–Si4 and Ss1–Ss4 are from experiment SOGASEX (4.5 m). See text for details. The quantities listed are as follows: $N =$ number of consecutive 30-min data; $H_u =$ measured significant wave height (m); $H_{\text{sd}} =$ swell–significant wave height (m) [cf. Eq. (4)]; $u_\text{aw} =$ friction velocity (m s$^{-1}$); $U_{10} =$ mean wind speed at 10 m (m s$^{-1}$); $n_p =$ swell peak frequency (s$^{-1}$); $\kappa_p =$ swell peak wavenumber (m$^{-1}$); $c_p/U_{10} =$ wave age ($c_p =$ swell peak phase speed); and $L_{\text{MO}} =$ the Obukhov length [Eq. (1)].
Figs. 2 and 3, although the magnitudes of the respective peaks vary widely. To be able to make a meaningful comparison of data from the four sites, they must, however, be referred to the same height, the actual measurement heights varying from 2.6 (BASE) to 8 m (in Rieder and Smith 1998). One of the datasets (RED) has simultaneous measurements at four levels, which enables a systematic study of the vertical structure of the marine surface layer and, hence, offers a tool for converting the data from all the sites to a common height and also gives an insight into the mechanisms involved.

a. Variation of RED spectra with height

To illustrate the relevant features, data from RED case RC have been used. Figure 4a shows, for the vertical component $nS_w(n)$, the mean energy spectra on a linear scale against frequency $n$ (Hz) on a logarithmic scale. The figure displays the spectra at different heights—5 (blue), 7 (red), 10 (green), and 14 m (black)—for the four measuring levels during RED case RC. A cospectrum, case RC, is also plotted in Figure 4b, which further illustrates the variability of the spectra with height.

**Fig. 4.** (a) Vertical velocity spectra $nS_w(n)$ plotted against frequency $n$ for the four measuring levels—5 (blue), 7 (red), 10 (green), and 14 m (black)—during RED case RC. (b) As in (a), but for the $u-w$ cospectrum, case RC.
scale. As already noted, this representation is area preserving (i.e., spectral energy within a given spectral band is proportional to the area under the curve). The corresponding wave spectra show a peak at around 0.12 Hz. A very strong peak close to this frequency is found also in the longitudinal and in the lateral spectra at all heights (not shown). The amplitude of the peaks decreases gradually with increasing height.

Figure 4b displays the corresponding mean $u$–$w$ cospectra, which also show a strong spectral peak at around the swell frequency, with amplitude that decreases with height. Note that this peak is negative (i.e., it contributes to the stress at the surface). A corresponding peak in the $v$–$w$ cospectrum also contributes to the stress magnitude [Eq. (2)].

Plots of second-moment statistics for case RC (not shown) reveal that $\bar{u}^2 w^2$ and the vertical variance $\sigma^2_w$ are nearly height constant. The corresponding variance for the longitudinal and lateral components $\sigma^2_u$ and $\sigma^2_v$ both decrease with height but only by about 10% from 5 to 14 m. From Figs. 4a,b, it is clear that the relative part of the respective spectra, which is taken up by the swell peak decreases rapidly with height. Nevertheless, the total variance in the $w$ component and in $\bar{u}^2 w^2$ is close to height constant. This means that peak energy is being gradually transferred to turbulence at a wide range of frequencies with increasing height.

The process can be studied very clearly in the plot for the vertical velocity (Fig. 4a). Graphical evaluation shows that the magnitude of the swell peak at 5 m takes up about 40% of the total spectral energy at this height, but it is seen to decrease rapidly with height. At the same time, as noted above, observations clearly show that $\sigma^2_w$ varies very little with height. One may have expected the swell energy to be cascaded down to smaller scale at higher levels. Surprisingly enough, Fig. 4a shows that the opposite occurs with energy instead increasing gradually with height in the spectral range $10^{-3} < n < 6 \times 10^{-2}$ Hz. This process also results in an increase of the cospectral level in the same frequency range (Fig. 4b). Further discussion of the mechanism behind the observed features is postponed till after the general analysis of the stress budget (section 5c).

b. Quantitative evaluation of swell cospectral peaks and their variation with height

It was earlier noted that the wave filter technique employed by Rieder and Smith (1998) effectively resulted in removal of the strong cospectral peaks in Figs. 2 and 3, giving the red curves in these plots. Thus, employing such a filter is also an effective way to identify these peaks. But it is also clear that the peaks can be identified by a straightforward graphic method, by drawing a subjective baseline, as indicated in Figs. 2 and 3.

The spectral peak shape can be approximated by that of a triangle, with its height equal to the spectral amplitude $[n_p C_{uw}(n_p)]_{\text{amp}} = [n_p C_{uw}(n_p)]_{\text{aperp}} - [n_p C_{uw}(n_p)]_{\text{baseline}}$ and its base equal to $\Delta \ln(n) \approx (n_2 - n_1)/n_p$, so that the spectral peak contribution

$$dP_{uw} = (n_2 - n_1) C_{uw}(n_p)/2 = n_2 - n_1 n_p [n_p C_{uw}(n_p)]_{\text{amp}}/2.$$  

(5)

For example, from Fig. 2 we get the following, approximately: $[n_p C_{uw}(n_p)]_{\text{amp}} \approx 2.2 \times 10^{-2}$; $n_1 = 6 \times 10^{-2}$ Hz, $n_2 \approx 0.16$ Hz, and $n_p \approx 0.085$ Hz, giving spectral peak energy $\approx 1.29 \times 10^{-2} \text{ m}^2 \text{ s}^{-2}$.

For the two cases from Rieder and Smith (1998) (Figs. 2 and 3), it is clear that the estimate of the spectral peak stress contribution obtained with the simplified graphical method is virtually the same as that obtained after applying the spectral filter method [the difference of the area under the original (blue) curve and the filtered (red) curve]. Since the cospectral peaks at dominant swell frequency are typically equally easy to identify as those in Figs. 2 and 3—although the amplitude may vary over a wide range—we chose the graphical method for evaluation of its magnitude throughout the entire dataset. An exactly analogous method is employed to estimate the magnitude of the $vw$-cospectral peaks.

The $u$–$w$-cospectral peaks were evaluated for all four measuring heights (5.1, 6.6, 9.9, and 13.8 m) for each of the seven RED cases (RA–RC and RE–RH of Table 1). For each case, the spectral peak values were plotted on a logarithmic scale against height on a linear scale. In this representation, all plots indicate that the corresponding data points appear to scatter around a straight line (see Fig. 5, which shows four examples). This implies that the magnitude of the peaks $S(z)$ decreases exponentially with height (cf. Makin and Mastenbroek 1996):

$$S(z) = S(0) \exp(-A \kappa_p z),$$  

(6)

where it has been assumed that the height scales with the swell wavenumber $\kappa_p$, and $A$ is a dimensionless parameter to be determined. Concerning the values of $A$, the following result was obtained:

<table>
<thead>
<tr>
<th>Case</th>
<th>RA</th>
<th>RB</th>
<th>RC</th>
<th>RE</th>
<th>RF</th>
<th>RG</th>
<th>RH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>2.1</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.1</td>
<td>1.7</td>
<td>2.0</td>
</tr>
</tbody>
</table>

To find out if the observed variation of $A$ is governed by any bulk atmospheric or wave-state parameter, $A$ was plotted, in turn, against each of the following parameters: $U_{10}$, $H_{sw}$, $\lambda$ (swell wavelength), $H_{sw}/\lambda$, $\kappa_p/U_{10}$, and
(c_p - U_{10}) (not shown). As it was not possible to conclude that A correlates with any of these parameters, it was a reasonable choice to take the mean of the seven A values (A = 1.43) to be representative for the RED data and, as a best guess, for the entire dataset in Table 1. As will be seen later (section 5a), the sensitivity of the final result to the value of A is evaluated by carrying out calculations for three alternative A values for each case (A = 1.0, 1.43, and 2.0, respectively), thus covering the entire range of variability of A in the RED data. The calculations, however, turn out to be fairly insensitive to the exact value of A. Makin and Mastenbroek (1996) quote earlier thesis work by Makin, who carried out “2D numerical calculations based on a mixing length theory” to parameterize the turbulent stress, which gave A = 2. They also conclude that for “short waves which support most of the surface stress, A = 5.” Thus our finding that A = 1.43 ± 0.5 appears not unreasonable.

4. The stress budget at the surface

a. Defining the stress

The horizontal stress, or drag, is a vector \( \mathbf{\tau} \), the magnitude of which is obtained with Eq. (2). In the atmospheric surface layer, this vector is mostly assumed to be colinear with the mean wind vector. Orienting the x axis along the mean wind direction then implies that the crosswind component of the stress is zero. In most cases, both over land and over sea, this is a reasonable approximation. In the case of swell-dominated conditions, there are, however, many reports on considerable off-wind stress angles [see, e.g., Rieder and Smith (1998) and further references in that paper]. Also in the present dataset, considerable off-wind stress angles are observed; in RED, values are typically in the range 30°–45°, but values up to 70° occur in some of the cases.

The horizontal along-wind and across-wind components of the equation of motion can be written, respectively, as follows:

\[
\begin{align*}
\frac{d\bar{u}}{dt} &= -\frac{1}{\rho} \frac{\partial P}{\partial x} + f \bar{v} - \frac{\partial}{\partial z}(\bar{u}'\bar{w}') \quad \text{and} \quad (7a) \\
\frac{d\bar{v}}{dt} &= -\frac{1}{\rho} \frac{\partial P}{\partial y} - f \bar{u} - \frac{\partial}{\partial z}(\bar{v}'\bar{w}'). \quad (7b)
\end{align*}
\]

where \( P \) is mean pressure, \( \rho \) is air density, and \( f \) is the Coriolis parameter.

As follows from Eqs. (7a) and (7b), in stationary and horizontally homogeneous conditions, \( \bar{u}'\bar{w}' \) and \( \bar{v}'\bar{w}' \) are expected to be close to height constant in the surface layer, where \( f \bar{u} \) and \( f \bar{v} \) are negligible. This is also supported by the RED data, as noted in section 3a.

b. The stress budget

The stress at the water surface is the sum of contributions from form drag and tangential drag. We formally write the stress budget as a sum of four terms (cf. Högström et al. 2009):

![Fig. 5. Examples of swell spectral peak amplitudes $S$ for the $u$–$w$ cospectrum $C_{uw}$ in RED plotted against height in a linear–logarithmic representation: case RG (black), RE (red), RH (green), and RC (blue).](image-url)
The first two terms on the right-hand side are both tangential drag terms: (i) the contribution from the swell peak and (ii) the remaining tangential drag. The last two terms are due to form drag; wave form drag gives downward momentum flux for waves moving slower than the wind [term (iii)]; for waves moving faster than the wind (i.e., for swell waves), the momentum flux is directed upward [term (iv), \((D_{\text{form}})_{\text{swell}}\) being negative (section 4d and appendix B)]. This means that the terms (i), (ii), and (iii) give positive contributions to the total stress, and the very last term gives a negative contribution.

In the introduction, it was described how Rieder and Smith (1998) applied a filter method to remove the influence from swell waves on the total measured stress. It resulted in a relation for the residual drag coefficient, attributed to the wind-sea waves, as a function of wind speed only. In section 4c below, we check how our values for \(u_{\text{tot}}^2 - (D_z)_{\text{swell}} - (D_{\text{form}})_{\text{swell}}\) give a residual drag coefficient, which is a linear function of \(U_{10}\).

The following two quantities have been plotted as function of the 10-m wind speed for all the 18 cases listed in Table 1: total drag coefficient \(C_D = u_{*0}^2 / U_{10}^2\) as circles; residual drag coefficient \(C_D_{\text{res}} = [u_{*0}^2 - (D_z)_{\text{swell}} - (D_{\text{form}})_{\text{swell}}] / U_{10}^2\) as triangles. The color code identifies the data source [RED (red), BASE (blue), and SOGASEX (black)], open symbols indicate slightly stable cases, and filled symbols indicate slightly unstable cases. Also shown in Fig. 6 is a solid line representing the expected drag coefficient relation for pure wind sea. The curve
is derived from a wind-sea dataset obtained during the FETCH experiment (Drennan et al. 2003), and this expression is seen to represent the residual data well and will be used later (section 6) in a surface stress model.

d. Evaluation of the swell form drag

As shown in detail in appendix B, analysis using the TKE budget to evaluate the swell form drag results in the following expression:

\[
(D_{\text{form}})_{\text{swell}} = \frac{T_p(0)\rho}{Bk_p c_p},
\]

where \(T_p(0)\) is the (extrapolated) value of the pressure transport term at the water surface, which is obtained from the respective TKE budgets, \(B\) is a constant = 1.9, \(\kappa_p = 2\pi/\lambda_p\) is swell peak wavenumber (see Table 1), and \(c_p = g/2\pi n_p\) is swell peak phase speed. Since \(T_p(0) < 0\) according to the measurements, \((D_{\text{form}})_{\text{swell}} < 0\) as well.

TKE budget analysis is required for direct determination of the quantity \((D_{\text{form}})_{\text{swell}}\), which, in turn, requires measurements at several levels, which are available only for the seven RED cases RA–RC and RE–RH and the single BASE case BA. These data will be used in an attempt to generalize the results.

A strict physical analysis would require \((D_{\text{form}})_{\text{swell}}\) to be properly made dimensionless and expressed as a function of some other dimensionless variable. It is reasonable to divide \((D_{\text{form}})_{\text{swell}}\) by \(u^2_{*}\). Thus, we search for an expression of the type \(y = -(D_{\text{form}})_{\text{swell}}/u^2_{*} = f(X)\), where \(X\) is some other dimensionless combination of relevant physical parameters, and \(y > 0\). Simple correlations of \(y\) with various parameters \([H_s, H_{sd}, U_{10}, \lambda, c_p, (c_p - U_{10})]\) shows that significant correlation is obtained only with significant swell wave height \(H_{sd}\) (Fig. 7).

Attempts to normalize \(H_{sd}\) with various parameters with dimension length \([U_{10}/n_p, (c_p - U_{10})/n_p, H_s, \lambda]\) was, however, not successful.

Although Fig. 7 is not dimensionally satisfactory, it will be used as an ad hoc description of the relation between \(y = -(D_{\text{form}})_{\text{swell}}/u^2_{*}\) and \(H_{sd}\). Linear regression valid for the range \(0.5 < H_{sd} < 2.0\) m is

\[
y = 0.269 - 0.126H_{sd}, \quad \text{for} \quad 0.5 < H_{sd} < 2.0\, \text{m}, \tag{11a}
\]

\[
y = 0 \quad \text{for} \quad H_{sd} > 2.0\, \text{m}. \tag{11b}
\]

In the dataset of Table 1, there are no cases with \(H_{sd} < 0.5\) m, but they are indeed common in the BASE dataset. But those cases have either swell peaks so small that they cannot be distinguished from the noise, or they have \(U_{10} < 3\) m s\(^{-1}\).

5. The swell influence on the surface stress

a. Results from the measurements

In section 3b, we evaluated the swell contribution to the surface stress from graphical estimation of the magnitude of cospectral peaks in the individual \(nCo_{uv}\) spectra. Since
the actual measurements are from various heights above the water surface, Eq. (6) is used to reduce the results to the water level $z = 0$. In this manner, we get a dataset of $dP_{\text{tot}} = (D_r)_{\text{swell}} + (D_{\text{form}})_{\text{swell}}$.

We seek a relation between $(D_r)_{\text{swell}} = dP_{\text{tot}} - (D_{\text{form}})_{\text{swell}}$ on one hand and some parameter related to swell wave characteristics on the other. Figure 8 shows that very good correlation is obtained between $(D_r)_{\text{swell}}$ and the parameter $H_{\text{sd}}^2n_p^2$. The line drawn corresponds to a linear relation:

$$ (D_r)_{\text{swell}} = 1.25 H_{\text{sd}}^2 n_p^2. \quad (12) $$

It is clear that the data points from the experiments RED, BASE, and SOGASEX all scatter around this line. The only outlier is the RED data point at $H_{\text{sd}}^2 n_p^2 = 0.026$, which is the case RA. As discussed in a forthcoming paper, the RA data belong to a different category than the remaining data of the present study.

The parameter combination $H_{\text{sd}} n_p$, which has the dimension of velocity, is proportional to the magnitude of the orbital motion $c_0$ at the water surface (cf. Fig. 9). This follows if we assume that the swell energy is concentrated in a narrow frequency band; then we can approximate the swell by a harmonic with the frequency $n_p$. According to the linear wave theory, the horizontal component of the orbital velocity at the surface in deep water will then be

$$ u_s = a \omega_p \cos(k_p x - \omega_p t), \quad (13) $$

where $a$ is the wave amplitude, $\omega_p = 2\pi n_p$, and $k_p = 2\pi/\lambda_p = \omega_p^2/\rho g$.

As seen from Eq. (13) and Fig. 9, the horizontal component of the orbital motion is equal to $-a\omega_p$ in the wave troughs and $+a\omega_p$ on the wave crests (i.e., $c_0 = a\omega_p$) and, since $a = H_{\text{sd}}/2.8$, we have $c_0 = 2\pi n_p H_{\text{sd}}/2.8$.

In section 3b, it was demonstrated that the spectral peaks vary exponentially with height [Eq. (6)]. Analysis of the cases from RED indicates that the parameter $A$ of this expression varied in the range $1.0 < A < 2.0$. Attempts to correlate $A$ with other bulk parameters were unsuccessful. Therefore, the data points in Fig. 8 were reduced from their respective measuring height to $z = 0$ with Eq. (6) and $A$ set to its mean value $1.43$. Alternative calculations were carried out with $A = 1.0$ and 2.0, respectively, but the effect on the result displayed in Fig. 8 is marginal (not shown).

A reviewer suggested that “misalignment of wind and swell” would lead to the observed downward momentum of swell. This hypothesis is tested on our data in Fig. 10, where $(D_r)_{\text{swell}}$ for all our data is plotted against the corresponding angle between the wind and the swell propagation direction. The plot indicates no correlation. This is in agreement with the prediction of the model presented in appendix A as a consequence of the fact that the form stress must be evaluated in a coordinate system that follows the wave and that the result is predicted to be rather insensitive to the angle of wind and swell propagation direction.

b. Physical interpretation of the swell stress/orbital motion plot

The airflow very close to the water surface is restricted by the fundamental no-slip condition, implying that the air is bound to follow the surface orbital motions of the water. This implies that a very thin layer of air close to the surface is dragged to and fro in an oscillatory motion. If we assume that the swell energy is concentrated in a narrow frequency band, we can approximate the swell by a harmonic with the frequency $n_p$. If the harmonic wave has the same energy as swell with significant wave height $H_{\text{sd}}$, the oscillatory motion of the surface is bound to cause a corresponding oscillating local boundary layer very close to the water surface. As seen from Eq. (13) and Fig. 9, the horizontal component of the orbital motion is equal to $-a\omega_p$ in the wave troughs and $+a\omega_p$ on the wave crests. Thus, introducing a coordinate system that follows the orbital motion at the surface and assuming that the phase-averaged mean wind profile in a local boundary layer of depth $\delta_1$ is logarithmic, gives a “velocity defect law” (cf. Monin and Yaglom 1973, Ch. 5.5) in the wave troughs:

$$ \frac{c_0 - \overline{u}(z)}{u_{*1}} = -2.5 \ln(z/\delta_1), \quad z < \delta_1, \quad (14a) $$

where $\overline{u}(z)$ is the local wind perturbation caused by the orbital motion at height $z$ above the water surface, and
$u_1$ is the corresponding friction velocity. For the wave crests, we get, in a corresponding manner,

$$
\frac{c_0 - \tau(z)}{u_2} = 2.5 \ln\left(\frac{z_0}{\delta_2}\right), \quad z < \delta_2,
$$

where $u_2$ is a new friction velocity, which may differ from $u_1$ and $\delta_2$, the depth of the local boundary layer over the crest, which may also differ from the corresponding value for the trough $\delta_1$.

The height $z$ where $\tau$ in Eq. (14a) is zero we define as the local roughness length $z_0_1$:

$$
\frac{c_0}{u_1} = -2.5 \ln\left(\frac{z_0}{\delta_1}\right). \quad (15a)
$$

In analogy, at a height $z = z_0_2$ where $\tau$ in Eq. (14b) is zero, we have

$$
\frac{c_0}{u_2} = 2.5 \ln\left(\frac{z_0}{\delta_2}\right). \quad (15b)
$$

An approximation of the mean friction velocity caused by the orbital motion for an entire wave period may be obtained as the average of $u_1$ and $u_2$:

$$
\bar{\tau} = (u_1 + u_2)/2
$$

Note that Eqs. (15a) and (15b) have opposite signs so that $u_1$ is negative and $u_2$ positive. This, in turn, implies that, in order for $\bar{\tau}$ to be nonzero, there must be a systematic asymmetry so that $z_0_1/\delta_1 \neq z_0_2/\delta_2$. Such an asymmetry is, in fact, to be expected. Equation (13) implies that the water is moving at the highest horizontal velocity at the crest, and therefore particles at the crest are catching the particles that are in the forward side of the wave and escaping the particles on the rear side (i.e., the water is converging at the crests and diverging at the troughs). When there are short waves riding on the top of the swell, the wavelength of the riding waves will be shortened at the crest, and they become steeper there. At the troughs, the riding waves will become longer. Such action is likely to affect both surface roughness $z_0$ and the depth of the local boundary layer $\delta$. If indeed this is the case, we would expect the following relation to hold:

$$
(D_\tau)_{swell} = \frac{u_2^2}{(D_\tau)_{form}} = \frac{\alpha c_0^2}{\beta H_\tau^2 n_2^2}, \quad (17)
$$

where

$$
\alpha = \left[\frac{1}{-2.5 \ln(z_0_1/\delta_1)} + \frac{1}{2.5 \ln(z_0_2/\delta_2)}\right]^2/4. \quad (18)
$$

From general physical considerations alone, we can say nothing more about the factor $\beta$ other than that it is likely to be nonzero. But the observations (Fig. 8) give $\beta = constant = 1.25$. Requiring that the energy of the

![Fig. 9. Schematic of the motion of the water particles under a harmonic wave that moves from left to right in deep water. Arrows at the water surface in the wave trough indicate a surface orbital velocity $= -u_1$; the corresponding value at the wave crest is $+u_2$. The surface is diverging at the trough and compressing at the crest. Therefore, the wavelength of a steep short wave riding on the top of the long wave will be shortened at the crest and lengthened at the trough.](image)

![Fig. 10. The logarithm of the quantity $\tau = (D_\tau)_{swell} - (D_\tau)_{form}$ plotted against the difference between the wind direction and the swell propagation direction: RED (blue asterisks), SOGASEX (black circles), and BASE (purple triangle).](image)
harmonic wave $\rho ga^2/2$ is the same as the energy of swell
$\rho gH_{sw}^2/16$, we get $a = H_{sw}/2.8$, which gives the relation
$\beta = (2.24)^2a = 5a$.

c. Discussion of the mechanism for the momentum flux

In section 5b, it was demonstrated how the peaks in the $u$–$w$ cospectra are likely to arise as a result of a combination of the orbital motions in the water and a systematic asymmetry in the structure of the ripples on the surface of the long waves. This combines into a net pulse of downward momentum flux in a frequency band near the swell peak frequency, as observed in Fig. 4b. At that same frequency, the vertical displacement of air over the swell waves also creates a flow perturbation in the atmosphere that propagates upward from the surface, carrying with it the downward momentum flux created by the surface process described above.

Figures 4a and 4b demonstrate clearly that, in the layer of atmospheric measurements (5–14 m), there is a transfer of kinetic energy and momentum flux in the direction from wave- (i.e., swell) induced perturbations to turbulence. Such effects have been shown to dominate the energy balance for the organized motions studied in Reynolds and Hussain (1972), Liu and Merkine (1976), Hsu et al. (1981), and Makin and Kudryavtsev (1999). In Rutgersson and Sullivan (2005), direct numerical simulations (DNS) over a moving wavy surface was studied, showing that, for fast waves, the underlying surface generates high turbulence levels, and this kinetic energy feeds the wave perturbation in the airflow. The strength and direction of the transport between wave kinetic energy and turbulent kinetic energy was shown to be sensitive to the size, phase, and tilt of the wave-induced turbulent Reynolds stress, and the orbital velocity was shown to play an important role. These simulated features are in qualitative agreement with the orbital motion asymmetry in the structure of the ripples on the surface of the long waves’ mechanism introduced in section 5b.

Analysis of the TKE budget for the RED cases (Högström et al. 2013) clearly shows that pressure transport is of great importance. Note that buoyancy and pressure transport terms both act in the vertical component of the TKE budget and give similar, but not identical, effects in the shape of the corresponding velocity spectra. A characteristic feature of the vertical velocity spectra in Fig. 4a is strong increase of low-frequency energy (i.e., for $n < 0.06$ Hz) with height but decreasing spectral levels for high frequency ($n > 0.2$ Hz).

6. A model for determination of the friction velocity during swell

Combining the results from sections 4 and 5, it is possible to set up a model for calculating $u_\ast$ during following and crosswind swell in near-neutral atmospheric conditions from knowledge of the following governing parameters: $U_{10}$, $H_{sd}$, and $n_p$. A formal analysis will be performed only for data in the approximate wind speed range $3.5 \text{ m s}^{-1} < U_{10} < 10 \text{ m s}^{-1}$. The case $U_{10} < 3.5 \text{ m s}^{-1}$ will be treated in a forthcoming paper. Data for $U_{10} > 10 \text{ m s}^{-1}$ have much larger scatter in the $C_D$ plots Fig. 6, than the bulk of the data. A further reason to restrict the formal analysis to the range $3.5 \text{ m s}^{-1} < U_{10} < 10 \text{ m s}^{-1}$ is that this enables the use of the remaining RED data, except the time periods already used (cases RA–RC and RE–RH, Table 1) in a truly independent model test. After screening the original RED dataset for swell ($c_p/U_{10} > 1.2$) and removing data for cases RA–RC and RE–RH, there remain 399 half-hour data, which all fulfill the criterion $3.5 < U_{10} < 10 \text{ m s}^{-1}$.

The equation for the stress budget at the surface was given in Eq. (8). We further have, from Fig. 8, an expression [Eq. (12)] for $(D_\tau)_{\text{swell}}$:

$$ (D_\tau)_{\text{windsea}} + (D_\tau)_{\text{form windsea}} = (C_D)_{\text{windsea}} U_{10}^2, \quad (19) $$

where $(C_D)_{\text{windsea}}$ is obtained from Eq. (9).

Inserting Eqs. (12) and (19) in Eq. (8) and utilizing the expression $(D_\tau)_{\text{swell}}$ derived in section 4d gives

$$ u_\ast^2 = 1.25H_{sd}^2 n_p^2 + (C_D)_{\text{windsea}} U_{10}^2 - u_\ast^2 y. \quad (20) $$

where $y$ is calculated from Eqs. (11a) and (11b), respectively, for $0.5 < H_{sd} < 2$ m and $H_{sd} > 2$ m. Thus, the model equation becomes

$$ u_\ast = \sqrt{(C_D)_{\text{windsea}} U_{10}^2 + 1.25H_{sd}^2 n_p^2 \over 1 + y}. \quad (21) $$

Figure 11 shows $u_\ast$ derived with Eq. (21) from measured values of $U_{10}$, $H_{sd}$, and $n_p$, plotted against $u_\ast$ derived as the mean from measurements of $\overline{u'w'}$ and $\overline{v'w'}$ at 5 and 7 m with Eq. (21). For the dataset, consisting of 399 data, we get the following statistical measures:

$$ \text{mean}(u_\ast - u_\ast_{\text{meas}}) = 0.0084 \text{ m s}^{-1} \quad \text{and} $$

$$ \text{std dev}(u_\ast - u_\ast_{\text{meas}}) = 0.0185 \text{ m s}^{-1}, $$

which refers to the following range: $0.15 \text{ m s}^{-1} < u_\ast < 0.37 \text{ m s}^{-1}$. 


and the wind-sea curve [Eq. (9)] approach each other. From Fig. 12a, it is found that values of \( H_{sd}^2 \) are typically around 0.02 for RED, whereas in the SOGASEX experiment, values in the range 0.08–0.15 dominate. These results give indications of \( H_{sd}^2 \) values expected over the ocean.

Semedo (2010) used data from ERA-40 for the entire World Ocean to map down the distribution of various characteristics related to atmosphere–ocean interactions, with particular emphasis on swell. Thus, by combining information on “significant swell steepness” and swell–significant wave height, it is possible to derive estimates of \( H_{sd}^2 \). In a part of the thesis entitled “Global significant swell steepness,” Semedo (2010) employs “the significant swell steepness”

\[
sl_s = \frac{\sigma_s}{L_p},
\]

(22)

where \( \sigma_s \) is the standard deviation of the swell surface elevation, and \( L_p \) is the peak wavelength. Inserting the relation between wavelength and frequency \( L_p = (g/2\pi)(1/n_p^2) \) and noting that \( \sigma_s \approx H_{sd}/4 \), Eq. (22) becomes \( sl_s = H_{sd} 2\pi n_p^2/(g \times 4) \) so that

\[
H_{sd}^2 = \frac{g(4/2\pi)sl_s}{H_{sd}} \approx 6.4 sl_s L_p.
\]

(23)

Two maps in Semedo (2010) show “Seasonal averages of \( sl_s \) for Dec, Jan, Feb and June, July, Aug respectively.” They show that a very high percentage of the World Ocean has \( sl_s \) values in the range 2.5 \times 10^{-3}–3.5 \times 10^{-3} during both seasons. In Semedo et al. (2011) (Fig. 2 for December, January, and February and Fig. 3 for June, July, and August), the distribution of mean significant swell wave height shows typical values of \( H_{sd} \) for very large areas in the range 1.5–3 m. Taking \( sl_s = 3.0 \times 10^{-3} \) and \( H_{sd} = 2 \) m as typical values for the World Ocean, Eq. (23) gives \( H_{sd}^2 = 4 \times 10^{-3} \) as a typical value for the World Ocean. As seen from Fig. 13, this means substantial deviations from the traditional COARE relation. Note that this is a rough estimate of the mean value for the World Ocean, implying large regional and temporal variations in \( H_{sd}^2 \) and thus in \( C_D \) for a certain value of \( U_{10} \).

8. Conclusions

Data from three true oceanic experiments, RED, SOGASEX, and MBLEX (from Rieder and Smith 1998) and additional data from the Baltic Sea (experiment BASE) were used to study relations and mechanisms for the surface drag during swell conditions. The RED and MBLEX data were made with instruments mounted on the FLIP; SOGASEX and BASE data were
taken from measurements on ASIS buoys. All data are motion corrected. During RED, turbulence measurements were made at four heights above the water (5, 7, 10, and 14 m), which enables systematic studies of the variation of wave influence with height. General requirements are (i) $c_p/U_{10} > 1.2$; (ii) near-neutral atmospheric stratification (i.e., $z_m/L_{MO} < 0.1$); (iii) the angle between the wind and the swell propagation direction $|\phi| \leq 90^\circ$, which means wind-following swell and crosswind swell (not counter swell); and (iv) reasonably stationary conditions during a number of consecutive 30-min periods ($2 < N < 16$). The datasets studied have $3.5 \text{ m s}^{-1} < U_{10} < 10 \text{ m s}^{-1}$.

The most remarkable and consistent finding of this study is that a negative peak of varying magnitude is found in all $u-w$ cospectra over swell, centered at the frequency of the dominant swell $n_p$, thus implying a net contribution to the downward-directed flux of momentum. Note that this is contrary to the situation much studied recently of upward-directed momentum flux due

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**Fig. 12.** (a) Histogram showing the frequency of $H_{sd,p}^2$ values in RED. (b) As in (a), but for the SOGASEX data.
...the capillary wave field in the wave troughs, where the water particles move toward the direction of the wave on one hand, and on the wave crests, where the water particles move in the opposite direction on the other.

Combining mathematical expressions for the following terms of the surface stress budget, 1) the downwind-directed swell peak effect, 2) the upward-directed momentum flux due to swell form drag, 3) the residual surface drag, parameterized as $(C_D)_{\text{windsea}} U_{10}^2$, enables a predictive expression for the friction velocity, which can be tested against measurements. This was done for the entire swell dataset from RED, except the 55 data included in cases RA–RH, resulting in 399 independent 30-min data that fulfill the criteria (i)–(iii) listed earlier in section 2a. The result is quite satisfactory: for the range $0.13 < U_{10} < 0.37 \text{ m s}^{-1}$, the main difference between predicted and measured $u_*$ is about 0.02 m s$^{-1}$, and the corresponding standard deviation is 0.027 m s$^{-1}$.

The above analysis indicates clearly that, in following and crosswind swell conditions in the range $3.5 < U_{10} < 10 \text{ m s}^{-1}$, $C_D$ is not only a function of $U_{10}$ as traditionally assumed, but it is also strongly dependent on the parameter combination $H_{sd}^2 n_p^2$. Statistics from the RED and SOGASEX experiments show that $H_{sd}^2 n_p^2$ is typically in the range 0.02–0.15. As illustrated in the parameter test plot in Fig. 13, this means that $C_D$ can easily attain values that are 2–5 times that predicted from standard wind speed–only parameterizations. At low winds, the no-swell curve is significantly below the COARE 3.0 curve, indicating that swell is present for much of the COARE 3.0 measurements. For higher winds, the wind-sea curve approaches the COARE curve.

Global data from ERA-40 studies (Semedo 2010) indicate that a typical mean value of $H_{sd}^2 n_p^2$ for the World Ocean is about 0.04. Since areas with $U_{10} < 10 \text{ m s}^{-1}$ are widespread over the ocean, it is expected that surface stress calculations are affected by the swell term $H_{sd}^2 n_p^2$ to a considerable degree over the ocean.

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APPENDIX A

Effect of the Angle between Swell Propagation Direction and Wind

Here it will be demonstrated that the effect of the angle $\phi$ between the wind direction and direction of swell propagation on the structure of the marine boundary...
FIG. A1. Schematic picture of wind–swell geometry for a case with monochromatic swell waves propagating toward the west with speed $c$ for a case with wind–wave angle $\phi \leq 90^\circ$. Vectors refer to a coordinate system moving with the wave: $c$ = wave phase velocity; $U$ = wind velocity; and $R$ = resultant flow vector.

layer is small for $\phi \leq 90^\circ$. The interaction between the wind and the waves is obtained from the relation for the form stress or pressure drag:

$$ (D_p)_0 = \frac{1}{\lambda} \int_0^\lambda p \frac{d\eta}{dx} dx, \quad (A1) $$

where $\eta$ is the height of the wave surface at position $x$, and $p$ is the pressure on the surface at $x$. In the case of a stationary wave (i.e., $c = 0$), the effective wave slope becomes $(d\eta/dx)(\cos \phi)$. But for the case with a finite wave speed, Eq. (A1) must be evaluated in a coordinate system that follows the wave. This is illustrated schematically in Fig. A1, where we consider a train of monochromatic two-dimensional waves traveling toward the west (i.e., to the left in the figure) with phase speed $c$. The relative motion of air and wave for $\phi \leq 90^\circ$ (following swell) is illustrated schematically in Fig. A1. Here, $c$ is assumed to be larger than $U$. The following vectors are shown: (i) the relative motion of the wave surface and the air that would ensue for the case of zero wind (i.e., a vector $c$ opposite to the wave motion and of the same magnitude $c$ as that of the wave phase speed; (ii) the wind vector $U$; and (iii) the resultant sum of vectors (i) and (ii) $R$.

The following relation is valid for the angle $\alpha_1$ between the resultant trajectory vector $R$ and $c$:

$$ \alpha_1 = \arctan[U \sin \phi/(c - U \cos \phi)]. \quad (A2) $$

From Eq. (A2), it can be evaluated how the angle $\alpha_1$, and hence the factor $\cos \alpha_1$, varies with wave age $c/U$ and the wind/wave angle $\phi$. It is found that, for $c/U > 2.0$, $0.86 < \cos \alpha_1 < 1$ for all values of $\phi$ (i.e., for practical purposes, $c/U \cos \alpha_1 \approx c/U$). For $c/U \approx 1.6$, the angle $\alpha_1$ varies somewhat with $\phi$, with $\cos \alpha_1$ having a minimum of 0.78 at $\phi = 45^\circ$.

The above result implies that, for large enough $c/U$ the resulting pressure drag $(D_p)_0$ [cf. Eq. (A1)] is largely independent of the wind/wave angle $\phi$. This result is valid for a gentle, symmetrical wave form (like a sine wave) that is not modified by the wind.

As mentioned in section 2, an alternative parameter to characterize the wave state is based on subdivision of the one-dimensional wave spectrum $S(n)$ into two parts, $E_{\text{swell}}$ and $E_{\text{windsea}}$, respectively. Analysis of the ratio $E_{\text{swell}}/E_{\text{windsea}}$ can thus be used as a wave-state parameter, as shown from measurements taken at the site Östergarnsholm in the Baltic Sea (Smedman et al. 2003). In Fig. A2a, the ratio $E_{\text{swell}}/E_{\text{windsea}}$ calculated from Östergarnsholm measurements is plotted against $c_p/U_{10}$ and in Fig. A2b against $c_p/(U_{10} \cos \phi)$. Figure A2a shows a strong relation between $E_{\text{swell}}/E_{\text{windsea}}$ and $c_p/U_{10}$, but this is not the case when $c_p/(U_{10} \cos \phi)$ is used.

The conclusion from this study is thus that, for mapping of the effect of wave age on the air–sea momentum exchange, the parameter $c_p/U_{10}$ should be used rather than $c_p/(U_{10} \cos \phi)$.

**APPENDIX B**

**Derivation of the Swell Form Drag Term**

As outlined in Högström et al. (2009), the form stress or pressure drag over a wave with wavelength $\lambda$ is

$$ (D_{\text{form}})_\text{swell} = \frac{1}{\lambda} \int_0^\lambda p \frac{d\eta}{dx} dx, \quad (B1) $$

where $\eta$ is the height of the wave surface at position $x$, and $p$ is the pressure at the surface at $x$. By writing

$$ \frac{d\eta}{dx} = \frac{d\eta}{dt} \frac{dt}{dx} = \frac{1}{c}, \quad (B2) $$
where \( w \) is the vertical velocity at the surface, and \( c \) is the wave phase speed, Eq. (B1) can be written as
\[
\frac{1}{\rho} \frac{\partial (\tilde{p} \tilde{w})}{\partial z} = T_p. \tag{B5}
\]
So, by inserting Eq. (B4) in Eq. (B5) and integrating from \( z = 0 \) to infinity, we get an expression for \( (\tilde{p} \tilde{w})_{z=0} \), which, with Eq. (B1), gives for the swell form drag
\[
(D_{\text{form}})_{\text{swell}} = -\frac{T_p(0) \rho_a}{B K_p \rho_c p}, \tag{B6}
\]
where \( c_p \) is the swell peak phase speed. This derivation from Högström et al. (2009) is for the budget of \( -u_w^2 \), so, in order to get the correct sign for our case [i.e., for the budget of \( u_w^2 \) (Eq. 8)], we have to change the sign in Eq. (B6) and finally get
\[
(D_{\text{form}})_{\text{swell}} = \frac{T_p(0) \rho_a}{B K_p \rho_c p}, \tag{B7}
\]
which is identical to Eq. (10) in section 4d.

REFERENCES


