Evolution of stable internal boundary layers over a cold sea

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Abstract. The situation studied in this paper relates to air flow from a heated land surface out over a water surface with much lower temperature. Observations from the Baltic Sea, where this kind of situation prevails for two thirds of the time, indicate that several distinct flow regimes occur. Simulations with a numerical boundary layer model for various combinations of geostrophic wind speed, $V_g$, and bulk temperature difference between land and sea, $\Delta \Theta$, have been performed. They show that, initially, stratification is always stable in the air layers near the sea surface but after some time transition to a near-neutral layer capped by an inversion takes place. The simulations, as well as the results of a simple analytical study, indicate that the transition takes place at a certain time of over water transport, $t_w$, which can be calculated when the bulk parameter $\Delta \Theta$ is known. Measurements of boundary layer temperature and wind profiles at three sites in the Baltic Sea area strongly support this result. Thus, for measurements representing traveling times less than $t_w$, derived with the above mentioned expression, a stable boundary layer is always found. For traveling times larger than $t_w$, a mixed layer is found (provided $\Delta \Theta$ is not large enough to cause quasi-frictional decoupling). Turbulence measurements made at the three sites simultaneously with the profiles show that $(u'V_g^2)10^3$ is between almost zero and 25 for the stable cases and between 25 and 46 for the mixed layer cases.

1. Introduction

In high latitudes, big water bodies surrounded by land masses are likely to develop a particular meteorological regime during a large part of the year, when warm air is advected out over the relatively much cooler surface of the water. Thus stable stratification, with correspondingly low turbulent exchange rates at the surface, is bound to occur. Earlier studies, referenced in section 2, have identified that the situation is likely to be still more complicated. During certain conditions the turbulent exchange is found to be almost nonexistent in spite of nonnegligible wind; during other conditions, no surface inversion exists and the exchange at the surface is quite large despite strong temperature contrast between elevated layers and the sea surface. This complex situation presents a serious problem for parameterization in numerical models, which may have great implication for, e.g., the water balance of a big lake or a semienclosed sea and the local climate of surrounding land areas.

The above situation has been found to apply to the Baltic Sea, which is now the focus of internationally coordinated research within the Baltic Sea Experiment (BALTEx), which is one out of five Global Energy and Water Experiments (GEWEX) regional experiments. The present study is based on measurements from several field stations in the Baltic Sea area and on simulations with a numerical meso-$\gamma$-scale atmospheric model. The purpose of the analysis is to develop a simple but effective scheme for identification of the different characteristic surface exchange situations that develop during the transport of warm air over a much colder water surface.

In section 2 the overall structure of the boundary layer over the Baltic Sea area is discussed. In section 3 the mesoscale model and the numerical simulations are described, and in section 4 a bulk stability parameter is introduced. Measurements from three sites, used to verify the simulations, are presented in sections 5 and 6.

2. Boundary Layer Over The Baltic Sea Area

The Baltic Sea is a semienclosed sea, which means that it is surrounded by land surfaces. The only outlet of water is through the Danish Straits and Öresund, the strait between Sweden and Denmark. As shown in Figure 1, the Baltic Sea is situated roughly between 55øN and 65øN. Here we will only treat the conditions within the Baltic proper, i.e., the part of the Baltic south of the Åland Islands (~60øN).

The boundary layer above a semienclosed sea like the Baltic Sea will almost always feel the presence of the surrounding land, that is to say advective effects will always influence the turbulence structure regardless of wind direction. The location of the Baltic Sea at fairly high latitudes means that the land surfaces will be warmer than the sea surface during a large part of the year, and thus stable internal boundary layers will build up over the sea. The difference in temperature between land and sea is largest during spring and early summer and can easily reach 15ø-20øC. During early winter the sea surface is usually warmer than the surrounding land.

There has been extensive research on internal boundary layers and the effects of abrupt changes in surface temperature from cold to warm but far less work on the offshore problem in terms of warm air flowing out over a colder sea. Stable internal boundary layers over a cold sea were first studied by Csanady [1974], who tried to estimate the shear stress exerted on the relatively cold surface of Lake Ontario. Mulhearn [1981] analyzed the formation of surface-base advective radar ducts based on dimen-
sional arguments, and the same approach was used by Gryning and Joffre [1987] and Melas [1989], who both studied the stable internal boundary layer over the Oresund Strait. Garratt [1987] and Garratt and Ryan [1989] applied a numerical model on aircraft data off the Australian coast and showed that the internal boundary layer could be characterized by a critical layer-flux Richardson number. Here a two-dimensional numerical model together with measurements is used to study the development of the stable marine internal boundary layer over the cold Baltic Sea.

2.1. Stratification

To give an indication of the frequency of stable stratification in the boundary layer over the Baltic Sea, 5 years of profile measurements of wind speed and temperature have been analyzed. The measurements were taken at the site Näsudden at the west coast of the island of Gotland (B in Figure 1). A 145-m-high tower is situated on a flat peninsula 1500 m from the coastline. There is an undisturbed over-water fetch for winds coming from 200° to 320°. When the wind is blowing from the sea, the internal boundary layer, building up over the land surface, can reach up to about 100 m for unstable stratification [Smedman and Högström, 1989; Bergström et al., 1988], but for neutral and stable conditions the internal boundary layer height is less than 70 m [Bergström et al., 1988]. To be able to estimate the stratification over the sea, the gradient Richardson Number ($R_i$) has been calculated for the height interval 96-145 m using three levels of wind speed and temperature measurements. $R_i$ is defined

$$R_i = \frac{\frac{\partial \Theta}{\partial z}}{\frac{T_0}{\partial z} \left(\frac{\partial U}{\partial z}\right)^2}$$

(1)
where $T_0$ is a reference temperature, $g$ is the acceleration of gravity, $U$ is the mean wind speed, $z$ is the height above ground, and $\Theta$ is the mean potential temperature. The measurements represent 30 min averages.

The calculated $R_i$ values have been represented in six stability classes from very stable ($R_i > 0.25$) to very unstable ($R_i < -0.25$). Figure 2 shows the distribution of stability classes for a height interval representing undisturbed sea condition, 75-96 m. As can be seen from Figure 2, stable stratification occurs during 66% of the time and with very stable conditions, $R_i > 0.25$, during about 35% of all time. As noted above, these $R_i$ values refer to the height interval 75-96 m above the ground. This means that they are likely to be representative for undisturbed upwind over-water conditions. The percentage figures are of course not directly transferable to conditions immediately above the water surface. What can be said is that the lowest 100 m or so of the marine atmosphere is probably stably stratified during as much as about two thirds of the year in this area (which is likely to be representative of the Baltic proper, i.e., the region south of the Åland Islands). This statement is valid at least in a bulk sense, allowing for possible existence of a shallow mixed layer immediately above the water surface during certain conditions.

### 2.2. Advective effects over the Baltic Sea

As stated above, advection of warmer air from the land surfaces surrounding the Baltic Sea will always affect the marine boundary layer. The turbulence structure of the boundary layer may change as the stable internal boundary layer slowly increases in height when warm air is advected out over cool water. However, Tjernström and Smedman [1993] observed that in spite of a large temperature difference between land and sea, the marine boundary layer sometimes is close to neutral. Garrett and Ryan [1989] also found stable internal boundary layers in off-shore flows. The initial internal boundary layer is stably stratified, but as advection distance increases, stability decreases and a well-mixed slightly stable layer capped by an inversion develops. However, they did not comment on this particular feature. Csanady [1974] also observed these conditions quite frequently over Lake Ontario. He showed that maintenance of a neutrally stratified mixed layer with an inversion at the top is possible for a certain type of flow. When warm air is advected out over the cool water, a stable internal boundary layer begins to form. At greater distances from the shore the inversion height converges to a constant height and the structure gradually becomes more and more independent of fetch. In this asymptotic equilibrium condition, heat flux from the air to the water must cease and the air close to the surface below the inversion therefore assumes the temperature of the water surface: neutral stratification. Brot and Wyngaard [1978] also find in their simulations the same evolution in time of the nocturnal stable boundary layer during the night. The aim of this paper is to derive a stability expression, as a function of external parameters only, which can be used to describe the evolution of the stable internal boundary layer over the sea.

### 3. Numerical Simulations of the Internal Boundary Layer

In order to support the idea proposed by Csanady [1974], simulations with a numerical boundary layer model have been performed. The model employed is the Department of Meteorology, Uppsala University model (MIUU-model), described by Enger [1990]. It is hydrostatic and has a second-order turbulence closure scheme, which is suitable for simulating internal boundary layers [Arritt and Physick, 1989]. Comparisons between numerical simulations and measurements during stable stratification show that the model is able to simulate stable boundary layer flow, provided the wind speed is not too low (R. J. Barthelmie et al., Observations and simulations of near-surface diurnal cycles of wind speed, submitted to Journal of Geophysical Research, 1996), (hereinafter referred to as Barthelmie et al., submitted manuscript, 1996). Because we want to study what happens when air from the heated land mass flows out over the cold water, a two-dimensional version of the model was employed.

The computations were carried out on a grid consisting of 60 horizontal and 20 vertical points. The horizontal distance between grid points was 2 km in the central area of the model domain, increasing telescopically toward the outer parts, with 26 km between the two outmost points at each end. In most of the simulations the coastline was located 117 km downstream of the first grid point (see Figure 3) with six grid points over land. Near the shore the distance between grid points was 8 km, decreasing to 2 km in the central model domain, where the transition of the flow structure is likely to occur (see below). The outmost grid point was situated 420 km from the shore line. In the runs with a geostrophic wind speed higher than 15 m s$^{-1}$ and with a temperature difference of 10°C the grid was changed so that the last grid point was situated 540 km from the coast. In the vertical the grid points were log linearly spaced, in order to give high resolution (1-2 m) close to the surface, while the resolution at the top of the model (2000 m) was about 200 m. In order to limit the vertical extent of the convective boundary layer over land, the vertical temperature profile in the upper part of the model domain was put to 4°C(Km)$^{-1}$, giving a boundary layer height around 400 m, which is typical for a high pressure situation at this latitude.

The roughness length was taken to be 0.1 m over land, which is a value typical for coastal areas in Sweden, and 0.0025 m over the sea. The model has been tested for different $z_0$ formulations including Charnock's formula (Barthelmie et al., submitted manuscript, 1996). In the range of geostrophic winds between 6 and 12 m s$^{-1}$ the use of the Charnock formula barley departed from the results with constant roughness.
The model was run for a number of combinations of temperature differences between land and sea (ΔΘ = 1.5, 4, 6.5, and 8 °C, see section 4) and geostrophic winds (Vg = 5, 10, 15, and 20 m s⁻¹). Figures 3a and 3b show cross sections of the simulated potential temperature over the sea, with a temperature difference between land and sea of around 6 °C and a geostrophic wind speed of 5 m s⁻¹ in Figure 3a and a geostrophic wind speed of 10 m s⁻¹ in Figure 3b. In both cases a stable internal boundary layer is being built up over the sea. Close to the shoreline the stratification is very stable, but as the advection distance increases the stratification decreases and finally becomes almost neutral. The same development can be seen in both figures but the advection distance to the point where a near-neutral boundary layer is found is much larger for an overlaying wind speed of 10 m s⁻¹ than for 5 m s⁻¹. In Figures 4a and 4b the same simulations are displayed but now as temperature profiles at different distances from the coast.

Figure 3. Simulated potential temperature (Kelvin) as a function of height and distance from the shoreline for a temperature difference between land and sea of about 6 K and with geostrophic wind speeds of (a) 5 m s⁻¹ and (b) 10 m s⁻¹.

Close to the shoreline the internal boundary layer is shallow and the temperature profile has a concave shape (negative curvature), which indicates a low degree of turbulence. But as the advection distance increases, the boundary layer height and the degree of turbulence also increase and the shape of the temperature profile changes from concave over a more or less linear shape to a neutral profile. The change in shape of the temperature profile indicates a gradual increase in turbulence which eventually will lead to a well-mixed boundary layer. The connection between the shape of the temperature profile and the relative importance of turbulent transport was first discussed by André et al. [1978] and André and Mahrt [1982]. Later, Stull [1983a, b] and Melas [1989] used the shape of the temperature profile to estimate the height of the stable boundary layer.

4. A Bulk Stability Parameter

This section provides a qualitative explanation for the behavior of the temperature profile considered here. When warm air is
flowing out over cold sea, the temperature of the air close to the
sea surface will be cooled due to turbulent heat transport, as
discussed above. But the sea surface temperature itself will be
almost unaffected because of the large heat capacity of the sea. Af-

ter some distance from the shoreline an equilibrium state is
gradually reached with a well-mixed boundary layer having an
inversion lid at the top, and the temperature profile will become
independent of distance. If we assume an analogy to the de-
velopment in time instead of distance, the change of temperature in
the boundary layer away from the shoreline can thus be approxi-
mated with the simplified diffusion equation

$$\frac{\partial \Theta}{\partial t} = \frac{1}{z} \frac{\partial}{\partial z} \left( K \frac{\partial \Theta}{\partial z} \right)$$

(2)

where \(K\) is the turbulent exchange coefficient for heat. Assuming
constant \(K\) the simplified solution of (2) can be written

$$\Theta_m - \Theta(z,t) = \left( \Theta_m - \Theta_{SST} \right) \frac{2 \sqrt{Kt}}{\sqrt{\pi}} \frac{\Theta_m - \Theta_{SST}}{\Theta_m - \Theta_{SST}}$$

(3)

where \(\Theta_m\) is the potential temperature above the internal bound-
ary layer and \(t\) will correspond to the transport time. A schematic
picture of the flow is shown in Figure 5. At the top of the bound-
ary layer the temperature is \(\Theta_m\), \(\Theta_{SST}\) is the surface temperature
over land, and \(\Theta_{SST}\) is the sea surface temperature. Over land, there is
a well-mixed boundary layer, which means \(\Theta_2 = \Theta_m\). The tem-
perature difference over land and sea at a specific time and height
can thus be written as \(\Delta \Theta = \Theta_m - \Theta_{SST}\). Although the square-root-
of-time dependency has been long known [e.g. Ertel, 1940], an-
other brief derivation is shown in the Appendix. The simplified
solution of (2), given in the Appendix, is valid for

$$z > \sqrt{Kt} > z/2$$

(4)

From the numerical simulations the height of the equilibrium lay-
and the transport time needed to reach the equilibrium \(t_e\)
can be estimated for all runs.

Brost and Wyngaard [1978] in their numerical simulations of
the stable boundary layer showed that the surface temperature de-
creased during the night as a function of the square root of time
until a uniform layer with near-neutral stability had been estab-
lished. Also Stull [1983a] discussed the (time)\(^{1/2}\) dependence for
what he called the “background cooling”.

The simplified analytical result, (3), and the analogue temporal
evolution over land during radiative cooling referred to above all
suggest that \(\Delta \Theta\) varies as the square root of the over-water trans-
port time. Figure 6 shows results from the simulations with the
MIUU model presented in the previous section. The model has been run for four geostrophic wind speeds (5, 10, 15, and
20 m s\(^{-1}\)) and for different temperature differences be-
tween land and sea. For each wind speed, four \(\Delta \Theta\) values were
given as input to the model (\(\Delta \Theta = 2.5, 5, 7.5, \) and 10°C). During
the initialization of the model the temperature over land changes
dependent on wind speed, which results in slightly different tem-
perature differences between land and sea for different wind
speeds. On the abscissa in Figure 6 is the temperature difference
\(\Delta \Theta\) divided by a reference temperature \(\Theta\) and on the ordinate is
the square root of the transport time \(t\) made nondimensional by
the Coriolis parameter \(f\). The Coriolis parameter is used just to
make the expression nondimensional, but the model has not been
run for different latitudes. The symbols are points taken from the
simulations representing the time of profile transformation from
stable to mixed state for different wind speeds. The transforma-
tion times are determined in a rather subjective way by looking at
individual profiles and selecting the time for “the first neutral
profile.” The data points all scatter around a line representing a
constant value \(-75\) for a nondimensional parameter \(S\) obtained by
dividing the ordinate of Figure 6 by the abscissa

$$S = \frac{1}{\sqrt{f}} \frac{\Delta \Theta}{\Theta}$$

(5)

Through the nondimensional parameter \(S\) it is possible to dis-
tinguish between an “ordinary” stable boundary layer \((S < 75)\)
and a mixed layer \((S > 75)\) from only external parameters: the
temperature difference between land and sea \((\Delta \Theta)\), the
geostrophic wind speed \((V_g)\), and the distance from the shoreline
\((X)\), \(t = X/V_g\). From (5) it is obvious that the stable boundary layer
over the sea can have different turbulent structures at different lo-
cations at the same time.

Figure 6. The square root of nondimensional transport time, for
reaching the equilibrium condition, as a function of nondimen-
sional temperature difference between land and sea. The values
are taken from simulations with different wind speeds: asterisks
5 m s\(^{-1}\); crosses 10 m s\(^{-1}\); circles 15 m s\(^{-1}\); plusses 20 m s\(^{-1}\). The
straight line gives the constant of proportionality, \(S = 75\).
Transport time and not distance being found to be the key parameter for the evolution of the stable boundary layer over the sea leads to the consequence that the time required to reach the thermal equilibrium is the same for all geostrophic wind speeds but will of course vary with temperature difference. However, the height of the equilibrium layer is a function of wind speed. A high wind speed generates large friction velocity and a more rapidly growing internal boundary layer, and thus the equilibrium height will be larger for higher wind speeds than for lower. The difference between equilibrium heights for geostrophic winds of 5 m s$^{-1}$ and 10 m s$^{-1}$ is clearly seen in Figures 4a and 4b. A low geostrophic wind speed generates a low degree of turbulence, but the equilibrium height is also low so the time to reach the equilibrium will be the same as for a high geostrophic wind speed, high degree of turbulence, and a much larger layer to mix to equilibrium.

However, the stability parameter $S$ can only be used to separate between a stable boundary layer and a well-mixed layer in a rather rough way, and it is not the intention here to get detailed information of the variation with time of either stability, friction velocity, or the internal boundary layer height.

5. Sites and Measurements

Figure 1 shows the locations of three sites A, C, and D in addition to site B, discussed earlier. At all three sites, profile and turbulence measurements have been performed. The towers are all situated close to the shoreline of low islands with a wide sector of undisturbed fetch over the sea. The internal boundary layers that will build up from the shore lines will never reach higher than 6 m at the position of the towers. Data from these sites consist of tower measurements of wind, temperature, and sometimes humidity profiles together with turbulence measurements at two or three levels. The turbulence instrument employed in two of the studies (A and D) is the MIUU instrument, which is described by Högeström [1982] and Bergström and Högeström [1987]. It is basically a wind vane-based three axial hot wire or hot film system supplemented with dry and wet bulb temperature sensors. The MIUU turbulence instrument has been corrected for flow distortion. At site C, in addition, three Solent Sonic instruments, which have been recalibrated in a large wind tunnel, have been used.

At sites A and C, extensive aircraft measurements have been performed over the sea outside the islands. The airborne measurements were taken with an instrument package mounted on a Sabreliner 40A aircraft. Wind was measured using the so called "radome gust probe" technique [Brown et al., 1983]. The aircraft measurement system is described by Tjernström and Frihe [1991].

The height of the different towers and the levels of measurements are given in Table 1. Radiosonde ascents (r), tethered balloon soundings (b), kite soundings (k), and pilib trackings (p) were sometimes performed to get the wind and temperature profiles up to 300-2000 m. Wave characteristics (w) were recorded at sites C and D.

All measurements were taken during stable stratification when the sea surface temperature was 5$^\circ$-20$^\circ$C colder than the land surface temperature. The experiment at site A (Uttängan) is described in detail by Tjernström and Smedman [1993] and the measurement campaign at site D (Nasskär) by Bergström and Smedman [1994] and Smedman et al. [1995].

Measurements from the first experiment performed at the site C (Östergarnsholm) are analyzed in two forthcoming papers.

During almost a week (May 30 to June 6, 1995) the synoptic situation over the Baltic Sea area changed very little. A weak high-pressure area was situated over northern Sweden, creating a geostrophic wind of 8-10 m s$^{-1}$ from southeast and a clear sky over the Gotland area. The sea surface temperature was 7$^\circ$C in the beginning and increased to 10$^\circ$C at the end of this time period. Temperatures over land in the Baltic states were around 20-25$^\circ$C in the middle of the day and around 15$^\circ$C during night. The transport time over water was between 6 and 10 hours. It was found that during this time period, the measured shearing stress and sensible heat flux were strongly suppressed compared to what would be expected from current theory (see next section). It is nevertheless found that turbulence is fully developed and continuous for most of the time. It is not clear if this turbulence regime may gradually develop into "an ordinary stable boundary layer" and a mixed layer or if the degree of turbulence remains so low that this will never happen irrespective of travel distance. We, however, coin the term "quasi-frictional decoupling" for this kind of flow regime, in order to distinguish it from both ordinary stable boundary layer flow and from cases with true frictional decoupling, which sometimes occur in situations with decaying surface waves [Yakovlev, 1970; Makova, 1975; Smedman et al., 1994].

6. Comparisons Between Measurements and Simulations

Tower data together with aircraft measurements around site A and radio soundings at site D are used to verify the numerical simulations. From the measurements the potential temperature profile and the geostrophic wind speeds are obtained. The geostrophic wind speeds are taken from the wind measurements at a height of about 2000 m. The temperature differences between land and sea are evaluated from weather maps, and the traveling distance over the sea is simply taken as the length of the straight trajectory, in the geostrophic wind direction, to the upwind coast. This is a rough way to estimate the transport time over the sea, but the aim of this investigation is to find some external parameters that can be used to characterize the turbulence flow structure over the Baltic Sea in a more general way.

Figure 7 shows $(\frac{g}{v})^{1/2}$ as a function of $\Theta_0$ from measurements. The circles indicate runs representing an equilibrium layer with a near-neutral temperature profile and crosses are runs with an ordinary stable boundary layer. Those measurements are from sites A and D. Very stable data from site C (Östergarnsholm) have been represented as the mean value with vertical and horizontal bars indicating the standard deviations. It can be seen from
Figure 7. The same plot as in Figure 6 but for measured data. The crosses indicate ordinary stable boundary layers and the circles indicate near-neutral mixed layers for sites A and D. Values from site C are indicated with a mean value and standard deviations (55 runs).

The figure that the line $S = 75$ divides the data sets according to stratification. In Figure 8, two examples of temperature and wind profiles are given. The mixed layer profile from site A (Utlångan) is shown in Figure 8a and a stable profile from site D (Näskär) in Figure 8b together with the calculated $S$ values.

The variation of the turbulent structure in the internal boundary layer with travel time over the water will cause a variation of the vertical exchange of energy and momentum over the sea. For example, in the stable boundary layer, heat and momentum fluxes are small and directed downward and in the mixed layer the heat flux is close to zero but the momentum flux can be considerable.

Figure 9 shows the measured geostrophic friction coefficient $(u_\ast / \sqrt{g})$ as a function of the parameter $S$ for the runs used in Figure 7. Again, crosses indicate stable boundary layers and the circles indicate equilibrium conditions from the site A and D, and the very stable profiles from site C are given as a mean value with standard deviations. For $S<75$, $(u_\ast / \sqrt{g})10^3$ ranges from less than 1 to maximum 25, but for $S>75$, $(u_\ast / \sqrt{g})10^3$ obtains values between 25 and 46. The very low geostrophic friction coefficients obtained for Östergarnsholm (mean value 8) are indications of a quasi-frictional decoupling regime.

7. Conclusions

One of the main objectives of BALTEX is to calculate turbulent fluxes over the Baltic Sea. To be able to do so, it is necessary to know the turbulence structure of the flow over the sea. In the present study, only stable stratification is dealt with. But long-term tower-based measurements from a site in the middle of the Baltic Sea proper show that the frequency of such conditions is as high as 66% on the average. These conditions are therefore deemed to be of fundamental importance for the climatology of the area.

When warm air is flowing out over the cold sea, a stable internal boundary layer is building up. Close to the shoreline the stratification is very stable and the turbulent exchange is weak. At larger traveling distances the inversion height converges to a constant height, and the structure gradually becomes more and more independent of fetch. In this asymptotic equilibrium condition the heat flux must gradually cease and a mixed layer will form and at the same time the momentum flux increases dramatically.

Both model simulations and some theoretical arguments indicate that the square root of the transport time up to equilibrium condition is proportional to the temperature difference between land and water. A stability parameter, $S$, defined only from external variables, can be used to distinguish between an ordinary stable internal layer and a near-neutral mixed layer. Measurements from two sites as well as model simulations confirm that $S = 75$ can be regarded as an estimate for the dividing line between the two.
Appendix

The diffusion equation neglecting the advection terms can be written

\[ \frac{\partial \Theta}{\partial t} = k \frac{\partial^2 \Theta}{\partial z^2} \quad (A1) \]

where \( k \), the exchange coefficient, is constant in time and space for an easy tractability and overall framework. Let \( \Theta(z,0) = \Theta_0(z) \) be the assigned starting profile and \( \Theta(0,t) = \Theta_{\text{SST}} \) be the sea surface temperature which is constant here. Introduce a new nondimensional variable

\[ \eta = \frac{z}{\sqrt{k}t} \]

Equation (A1) can thus be written

\[ \frac{d^2 \Theta}{d\eta^2} + \frac{\eta}{2} \frac{d\Theta}{d\eta} = 0 \quad (A2) \]

with the solution

\[ \Theta(z,t) = \Theta_{\infty} - \Theta_{\text{SST}} \sqrt{\frac{z}{\pi}} \int_0^\infty e^{-\eta^2/4} d\eta + \Theta_{\text{SST}} \quad (A3) \]

or

\[ \Theta_{\infty} - \Theta(z,t) = \left( \Theta_{\infty} - \Theta_{\text{SST}} \right) \left( 1 - \sqrt{\frac{I(z,t)}{\pi}} \right) \quad (A4) \]

where

\[ I(z,t) = \sqrt{\frac{z}{\pi}} \int_0^\infty e^{-\eta^2/4} d\eta \quad (A5) \]

Physically, the cooling of the air from below starts from time \( t = 0 \). This corresponds to arbitrarily large \( \eta \). As time and also the height increase (\( \infty \)), \( \eta \) may remain large enough to allow an expansion of \( I(\eta/\sqrt{kt}) \) for large \( \eta \):

\[ I = \sqrt{\frac{n}{\pi}} - \frac{\eta}{\eta_{\text{LARGE}}} e^{-\eta_{\text{LARGE}}^2/4} \quad (A6) \]

for \( \eta >> 1 \):

\[ I = \sqrt{\frac{n}{\pi}} - \frac{2}{\eta} e^{-\eta^2/4} \quad (A7) \]

and for \( \eta^2/4 < 1 \):

\[ \Theta_{\infty} - \Theta(z,t) = \left( \Theta_{\infty} - \Theta_{\text{SST}} \right) \frac{2}{\sqrt{\pi}} \frac{1}{z} \quad (A8) \]

The limits for \( \eta_{\text{LARGE}} \) are

\[ 1 < \frac{\eta^2}{kt} < 4 \quad (A9) \]

and finally

\[ \Theta_{\infty} - \Theta(z,t) = \left( \Theta_{\infty} - \Theta_{\text{SST}} \right) \frac{2 \sqrt{kt}}{\pi^{1/2}} \quad (A10) \]

as in the text, in (3). Having \( z > (Kt)^{1/2} > z/2 \) tells where the cooling is set in.

Acknowledgments. The field experiments were made possible by grants from the Military Weather Service of Sweden, the Swedish National Energy Administration under grant 506 224-4, and the Natural Science Research Council contracts G-GU 2684-302, G-GU 1775-300, and G-AA/GU 03556-313. Many thanks to all those who participated in the field experiments and especially to our technician K. Lundin. Ongoing fruitful discussions with U. Högström are also acknowledged.

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(Received March 8, 1996; revised August 25, 1996;
accepted September 4, 1996.)